DETERMINING THE THERMOPHYSICAL CHARACTERISTICS OF A MATERIAL LAYER WITH A UNIFORM TEMPERATURE FIELD

E. A. Belov, G. Ya. Sokolov, and A. S. Starkov

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A method is proposed for the processing of experimental data, by means of which it is possible to find the thermal conductivity and heat capacity of a plane layer. An automatic installation for nondestructive monitoring of heat-shielding coatings on metals is described.

One of the most important indicators of a class of thermophysical instruments is the productivity of the mesurement process, by means of which the duration of the preparatory and operating stages of the test are determined [1]. The majority of the methods of nondestructive monitoring of thermophysical characteristics (NDM TPC) with which we are familiar presupposes uniformity in the initial temperature field within the system formed by the measurement probe and the material being tested.

In this case, the time required to even out the temperature field is considerably greater than the time spent on the actual measurement [2]. The presence of a flow of heat in the background is characteristic of products being monitored under industrial and field conditions and, consequently, also characteristic is a temperature gradiant different from zero. Below, we propose a method for the determination of the thermal conductivity, volumetric heat capacity, and thermal diffusivity of a plane material layer with a constant temperature gradient. We analyze the thermal regime that is characteristic of a plane bicalorimeter, such as that used in devices to measure the thermal conductivity of heat-shielding coatings of the ITNR series [3, 4].

Let us examine an infinite plane layer of a material with constant physical characteristics and a linear distribution of temperature along an x axis perpendicular to the boundary surfaces (see Fig. 1). At the instant of time $\tau = 0$, a heat-conducting plate, superheated to 10-15 K, is melted on one of the surfaces. The temperature of the other boundary of this layer is kept constant and is subsequently taken as the origin of the calculation. The heat-conduction process in such a system is described by the equation

$$\frac{\partial^2 t(x,\ \tau)}{\partial x^2} - \frac{1}{a}\frac{\partial t(x,\ \tau)}{\partial \tau} = 0 \tag{1}$$

with the initial

$$t(x,\ 0) = \beta_1 x + \beta_2 \tag{2}$$

and boundary conditions

$$t(0,\ \tau) = 0, \tag{3}$$

$$\frac{\partial t(\delta,\ \tau)}{\partial x} = \frac{C_n}{\lambda}\frac{\partial t(\delta,\ \tau)}{\partial \tau}. \tag{4}$$

With ideal thermal contact between the plate and the surface of the layer, the temperature of the latter increases in a jump to values of $T_0 = t(\delta,\ 0)$. We will introduce into our examination the so-called suspended moments of temperature, i.e., integrals of the form [5]

$$\tilde{t}_k(x) = \int_0^\infty \tau^k t(x,\ \tau)\, d\tau, \tag{5}$$

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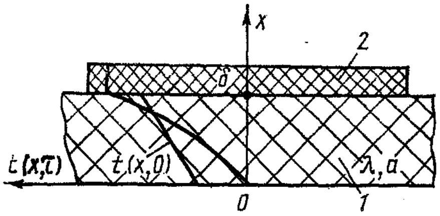

Fig. 1. Diagram of the heat-test experiment: 1) material being tested; 2) heat-conducting plate.

where k = 0, 1, 2, ... ellipsis.

Integrating Eq. (1) over time, for the function $\bar{t}_k(x)$ we obtain the following ordinary differential equation:

$$\frac{d^2\tilde{t}_k(x)}{dx^2} = \frac{1}{a} f_k(x), \tag{6}$$

where

$$f_k(x) = -\begin{cases} \beta_1 x + \beta_2, & k = 0, \\ k\tilde{t}_{k-1}(x), & k > 0. \end{cases} \tag{7}$$

Analogously, instead of condition (4) we find

$$\frac{\partial \tilde{t}_k(\delta)}{\partial x} = \begin{cases} \frac{C_n}{\lambda} T_0, & k = 0, \\ \frac{C_n}{\lambda} \tilde{t}_{k-1}(\delta), & k > 0. \end{cases} \tag{8}$$

The solution of Eq. (6), with consideration of (7), is expressed by a polynomial whose degree depends on the value of k. Let us note that, in accordance with (3), the free term of the polynomial must be equal to zero, so that

$$\tilde{t}_0(x) = \frac{1}{a}\left(-\beta_1 \frac{x^3}{3!} - \beta_2 \frac{x^2}{2!} + \beta_3 x\right), \tag{9}$$

$$\tilde{t}_1(x) = \frac{1}{a^2}\left(\beta_1 \frac{x^5}{5!} + \beta_2 \frac{x^4}{4!} - \beta_3 \frac{x^3}{3!} + \beta_4 x\right),$$

$$\tilde{t}_2(x) = \frac{2}{a^3}\left(-\beta_1 \frac{x^7}{7!} - \beta_2 \frac{x^6}{6!} + \beta_3 \frac{x^5}{5!} - \beta_5 \frac{x^3}{3!} + \beta_7 x\right),$$

$$\tilde{t}_3(x) = \frac{3}{a^4}\left(\beta_1 \frac{x^9}{9!} + \beta_2 \frac{x^8}{8!} - \beta_3 \frac{x^7}{7!} + \beta_5 \frac{x^5}{5!} - \beta_7 \frac{x^3}{3!} + \beta_9 x\right),$$

$$\tilde{t}_4(x) = \frac{4}{a^5}\left(-\beta_1 \frac{x^{11}}{11!} - \beta_2 \frac{x^{10}}{10!} + \beta_3 \frac{x^9}{9!} - \beta_5 \frac{x^7}{7!} + \beta_7 \frac{x^5}{5!} - \beta_9 \frac{x^3}{3!} + \beta_{11} x\right).$$

In order to find the coefficients β_3, β_5, ..., β_{11} we will successively substitute the equations of system (9) into condition (8). Assuming x = δ, we obtain:

$$\beta_3 = \frac{C_n}{c} T_0 + \beta_1 \frac{\delta^2}{2!} + \beta_2 \delta, \tag{10}$$

$$\beta_5 = \frac{C_n}{c}\,\frac{\lambda}{c}\tilde{t}_0(\delta) - \left(\beta_1 \frac{\delta^4}{4!} + \beta_2 \frac{\delta^3}{3!} - \beta_3 \frac{\beta^2}{2!}\right),$$

$$\beta_7 = \frac{C_n}{c}\left(\frac{\lambda}{c}\right)^2 \tilde{t}_1(\delta) - \left(-\beta_1 \frac{\delta^6}{6!} - \beta_2 \frac{\delta^5}{5!} + \beta_3 \frac{\delta^4}{4!} - \beta_5 \frac{\delta^2}{2!}\right),$$

$$\beta_9 = \frac{C_n}{c}\left(\frac{\lambda}{c}\right)^3 \tilde{t}_2(\delta) - \left(\beta_1 \frac{\delta^8}{8!} + \beta_2 \frac{\delta^7}{7!} - \beta_3 \frac{\delta^6}{6!} + \beta_5 \frac{\delta^4}{4!} - \beta_7 \frac{\delta^2}{2!}\right),$$

$$\beta_{11} = \frac{C_n}{c}\left(\frac{\lambda}{c}\right)^4 \tilde{t}_3(\delta) - \left(-\beta_1 \frac{\delta^{10}}{10!} - \beta_2 \frac{\delta^9}{9!} + \beta_3 \frac{\delta^8}{8!} - \beta_5 \frac{\delta^6}{6!} + \beta_7 \frac{\delta^4}{4!} - \beta_9 \frac{\delta^2}{2!}\right).$$

After substitution of Eqs. (10) into (9), we have a system consisting of five equations to determine the four unknowns λ, c, β_1, β_2. The fifth equation is a consequence of the first four, and it can be used for verification purposes. As an example, let us examine the case $\beta_2 = 0$, important from the practical standpoint, which is characteristic for studies into the thermal insulation layers on metallic bases in the operational process. In this case, we can limit ourselves to the first two equations from (9) and (10). After simple transformations, we obtain

$$\tilde{t}_0(\delta) = \frac{C_n T_0 \delta}{\lambda} + \frac{c}{\lambda}\,\frac{t_g \delta^2}{3}, \tag{11}$$

$$\tilde{t}_1(\delta) = \frac{2}{15} t_r \delta^4 \left(\frac{c}{\lambda}\right)^2 + \frac{1}{3} C_n T_0 \delta^3 \frac{c}{\lambda}\,\frac{1}{\lambda} + \frac{C_n \delta \tilde{t}_0(\delta)}{3},$$

where $t_g = \beta_1 \delta$ is the temperature of the surface for the material being investigated, prior to its installation on the bicalorimeter plate. System (11) can be solved for the two unknowns y = 1/λ and z = c/λ:

$$z = -\frac{B}{A} + \sqrt{\frac{B^2 - 4AD}{2A}}, \quad y = \frac{\tilde{t}_0(\delta)}{C_n \delta T_0} - \frac{t_g \delta}{3 C_n T_0} z,$$

where

$$A = \frac{1}{45} t_g \delta^4; \quad B = \left(1 - \frac{t_g}{T_0}\right) \frac{\tilde{t}_0(\delta)\,\delta^2}{3}; \quad D = \frac{\tilde{t}_0^2(\delta)}{T_0} - \tilde{t}_1(\delta). \tag{12}$$

Once we have calculated y and z, we find the thermal conductivity and the volumetric heat capacity of the material, namely, λ = 1/y and c = z/y.

Thus, to find the complex of thermophysical characteristics we must know the initial temperature of the surface t_g of the surface on the layer being investigated here, as well as the temperature moments $\tilde{t}_0(\delta)$ and $\tilde{t}_1(\delta)$ of this surface after it has been mounted on the bicalorimeter plate.

According to (5), the temperature moments can be determined during the regular stage of the thermal process, when the temperature of the bicalorimeter plate and that of the corresponding test-layer surface t(δ, τ) change exponentially at a constant rate m:

$$\tilde{t}_0(\delta) = \int_0^{\tau_1} t(\delta, \tau)\, d\tau + \frac{t(\delta, \tau_1)}{m}, \tag{13}$$

$$\tilde{t}_1(\delta) = \int_0^{\tau_1} \tau t(\delta, \tau)\, d\tau + \frac{t(\delta, \tau_1)}{m} \left(\tau_1 + \frac{1}{m}\right). \tag{14}$$

In a number of cases, in particular with studying elongated constructions, the direct determination of the quantity t_g is made difficult. In order to find this quantity, it is well to undertake a number of measurements on the overheating of the bicalorimeter plate relative to sme arbitrary temperature constant such as, for example, the temperature of that segment of the layer's surface outside of the zone of thermal perturbation. In this case, during the regular stage of the thermal process, the experimentally measured temperature $t_e(\tau)$ is related to t_g by the obvious relationship

$$t_g = \pm \{t(\delta, \tau_1) \exp[-m(\tau - \tau_1)] - t_e(\tau)\}. \tag{15}$$

The choice of the sign in (15) is determined by the direction of the background flow of heat in the layer. The quantities m and $t(\delta, \tau_1)$ in formula (15) can be found from the three values of the temperature $t_e(\tau)$, measured sequentially through a fixed interval of time Δτ. Indeed, it follows from (15) that:

$$m = \frac{1}{\Delta\tau} \ln \frac{t_e[\tau_1 + n\Delta\tau] - t_e[\tau_1 + (n+1)\Delta\tau]}{t_e[\tau_1 + (n+1)\Delta\tau] - t_e[\tau_1 + (n+2)\Delta\tau]}, \tag{16}$$

$$t(\delta, \tau_1) = \tag{17}$$
$$= \frac{t_e[\tau_1 + n\Delta\tau] - 2t_e[\tau_1 + (n+1)\Delta\tau] + t_e[\tau_1 + (n+2)\Delta\tau]}{t_e[\tau_2 + n\Delta\tau] - t_e[\tau_1 + (n+1)\Delta\tau]} \exp(mn\Delta\tau),$$

where $n = (\tau - \tau_1)/\Delta\tau$ is the sequential measurement number.

The relationships for the calculation of the temperature moments should also be expressed in terms of the experimentally measured temperature $t_e(\tau)$. For example, for the zero-th moment we find

$$\tilde{t}_0(\delta) = \int_0^{\tau_1} t_e(\tau)\,d\tau + t_g\tau_1 + \frac{t(\delta,\ \tau_1)}{m}. \tag{18}$$

The expression for the first moment is transformed in analogous fashion.

The validation of the proposed method was accomplished by means of an experimental installation based on a "DZ-28 Electronika" minicomputer. A late-model heating unit taken from a bicalorimeter for the nondestructive control of heat conduction, of the ITNR type described in detail in [3, 4], was used as the probe. The process of modernizing this equipment involved primarily the improvement of the information communications channels between the heating unit and the computer. In particular, in order to elevate the level of shielding against interference, the input amplification stages and the normalization of the signals from the primary temperature convertors, included among which were KD 520 A type semiconductor diodes, were located within the housing of the heating unit. To reduce the temperature drifting of the amplifiers, an internal circuit was introduced for the active control of the temperature in the elements of the input stage. The amplifier frequency range was limited to a level of 1 Hz by means of a Butterworth low-frequency filter to ensure the output of a constant useful-signal component against a noise background. This made it possible to translate the instantaneous analog information by means of high-potential (~10 V) signals, significantly to weaken the influence of pulsed interference and induction, characteristic of industrial conditions, thus reducing the intensity of fluctuations in the information channels of the installation.

In order to input data into the computer, we selected a regime with program interruption. With such a regime the input of the information can be accomplished without any rigorous time synchronization of the computer and the peripheral equipment. Proceeding on the basis that the "DZ-28 Electronika" computer is capable within a single operation of receiving only a single information byte, while the output of the information bus in the measuring device, for reasons of accuracy, was set at 16, the transfer of data to the computer was accomplished through a special linking unit. The latter changes the analog information from the heating unit into a series-parallel binary-decimal code and forms the control commands required for the exchange of information with the DZ-28. The algorithm for the processing of the experimental data includes smoothing out of the discrete values of the measured temperatures, analysis of the regularization of the temperature field within the material being tested, and the calculation of the sort characteristics with subsequent listing of the measurement results, the original data, and the basic calculation stages on the display of an SPP-15.

In order to test out the method, we undertook a number of control experiments on materials with known thermophysical characteristics. These materials included PS1-100 polystyrene and polymethyl methacrylate (PMMA) organic glass. the background flow of heat was generated by means of a wire heater installed on a metal plate, on which the specimen was mounted. The test results showed that in the background heat-flux density range of values below 100 W/m^2 the relative deviation in the experimental data from those cited in the literature [6, 7] does not exceed 6% for thermal conductivity and 8% for heat capacity.

NOTATION

x, coordinates; τ, time; $t(x, \tau)$, the temperature field within the layer of the material; T_0, initial superheating of the heat-conducting plate relative to the surface $x = 0$; λ, thermal conductivity; a, thermal diffusivity; c, volumetric heat capacity; C_n, heat capacity of the heat-conducting plate per unit area of contact with material; δ, thickness of layer; t_g, surface temperature of the investigated layer prior to installation on it of the bicalorimeter plate; m, plate cooling rate; β_1-β_{11}, constants.

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PRECISION METHODS OF NONDESTRUCTIVE CONTROL OF THERMOPHYSICAL PROPERTIES

V. P. Kozlov, V. S. Adamchik, and V. N. Lipovtsev

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We examine original solutions for two-dimensional problems of nonsteady heat conduction in the case of an orthotropic half-space with discontinuous mixed boundary conditions; these are used to develop new methods of controlling thermophysical properties in a nondestructive manner.

One of the most important problems confronting experimental thermophysics is the elevation of the accuracy and productivity of thermophysical measurements, convenience in their practical realization from a standpoint both of methodology and engineering. In comparison with many methods of determining thermophysical characteristics (TPC) in materials, greatest preference is presently given to nonsteady methods and means which ensure complex thermophysical measurements in various materials, without destruction of their natural structure and integrity (the so-called methods of means of nondestructive TPC control) [3-8]. The decisions of recent international and all-union thermophysical conferences on the properties of materials have recently stressed particular attention on the need for promising developments in the attainment of nondestructive control of TPC.

The proposed methods of nondestructive control (TPC) can be incorporated under the concept of precision methods, since the hypothetical boundary conditions are rather precise and can easily be achieved in actual practice.

In the area of measurement techniques involving nonelectrical quantities, the concept of a precision instrument (device) is constantly accompanied by the concept of the precision method. The metrological aspect of the need to unify these concepts is felt most urgently in the techniques of thermophysical experimentation. The methodological error in the measurement of thermophysical properties is determined primarily by the extent of divergence between theoretically postulated boundary conditions and those actually encountered in practice.

If the hypothetical boundary conditions are satisfied to a sufficient degree of accuracy during the course of a thermophysical experiment, we can render judgement as to the real accuracy of these thermophysical measurements, since only those that are associated with instrumental error in the determination of the TPC will be dealt with in our analysis of the errors. However, methods of reducing these errors are widely known and involve the utilization of perfected (precise) measurement capabilities to determine individual quantities included in the appropriate theoretical formulas for the determination of TPC. In the actual practice of thermophysical measurements, two forms of boundary conditions are most easily realized: constancy of temperature at the surface of the test body; constancy of the heat flow, provided that the latter is generated by means of a low-inertia electric heater

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